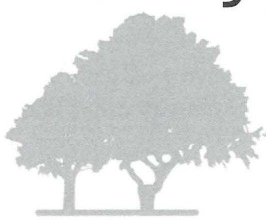
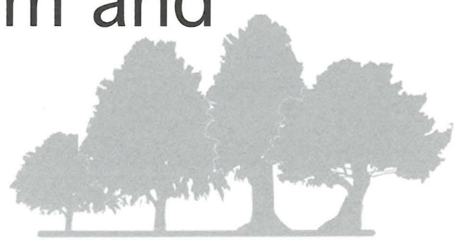
Forestry is Applied Mathematics Part 2 Pythagoras' Theorem and SOHCAHTOA





by **Andrew Weatherall**

I can tell it is only the second lecture in the first semester of first year for National School of Forestry students on their 'Measuring Trees and Forests' module because they are already all sat in the room when I arrive, right on time. Also, they hush as I enter. Neither of these things will last, thankfully, I find them slightly disconcerting.

"Talk among yourselves" I say, and they do, kind of, but not with the lack of care that second and third years would have. It is not that familiarity breeds contempt, but that it takes time for them to grow into the role of adult, and, sadly, fee-paying learners.

As soon as I move across from firing up the computer towards the whiteboard, I can feel their eyes on me. I know they are dreading the appearance of more scientific names of tree species to be tested on, but that is a challenge in another first year module.

'S O H C ...' I write, and the murmuring begins.

"Sine"' says one.

"Cosine" another adds, confirming at least a couple of them are on the right track.

"Opposite" returns the first.

'... A H T ...'

"Tangent" a chorus now.

"Adjacent", it is the first one again, probably someone who did well at GCSE Maths, possibly even did it at A level, maybe even would admit to liking maths. I had better watch out, they will almost certainly know more than I do.

'... O A'.

Finished I step back to survey the whiteboard with them:

S O H C A H T O A

"SOHCAHTOA"' I say. "Not the scientific name of an exotic tree species, so what is it?"

After previous experience of my lecturing style, they are wary of seemingly easy answers, but as the silence lengthens, one of them reluctantly ventures:

"SOHCAHTOA is a way of remembering the way to find angles in triangles."

"Correct"'

I know they are expecting a catch, but I tell them that the SOHCAHTOA mnemonic is to help them remember the trigonometrical ratios for right-angled triangles. I draw a couple of right-angled triangles, reminding them that 'H' is for hypotenuse, the longest side. We discuss Pythagoras and his theorem. We use:

$$a^2 + b^2 = c^2$$

with 3 and 4 unit length opposite and adjacent sides to work out that the hypotenuse is 5 units. A calculation that surely must be ingrained on the mind of every maths teacher

$$\begin{aligned} 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ c &= 5. \end{aligned}$$

We turn things around to show that with any two sides of a right-angled triangle we can find the third. I remind them about Archimedes and π, from the last session, and suggest that forestry is applied ancient Greek philosophy. There is not a single flicker of amusement. Then we look at how we only need the lengths of two sides of a right-angle triangle, or one side and a second angle, to work out the values of all the other angles and sides using SOHCAHTOA.

But, I deliberately do not go in to any further detail. This is all we need for the forestry lecture today.

"So", I ask, "if forestry is applied mathematics, how can we use SOHCAHTOA in a module called Measuring Trees and Forests?"

They stare at the whiteboard, uncertainly at the expectant look on my face, glance at their notes, stare at the whiteboard again, then one brave soul tentatively suggests:

"Could we use SOHCAHTOA to measure the height of a tree?"

The lecture has begun.

"Which bit would we use?" I ask.

They frown at the whiteboard.

"How many of you came to an Open Day or Applicant Day?" I ask, and about a third to a half put their hands up. It always astonishes me that not everyone visits us to confirm their choice of course and campus.

"Did you measure the height of the tree at the top of the hill?" Most of them nod.

Now they have a clue, I deliberately try not to mention SOHCAHTOA at recruitment events, but, weather providing, I like to take them out to measure tree diameter (using π) and height as an example of the kind of fieldwork they would do on our courses.

"We measured out the horizontal distance from the tree and used a ... a thing to calculate the height".

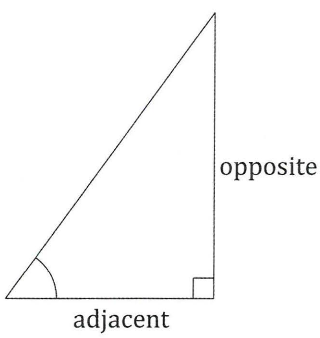
"A clinometer", I prompt. I really do not expect them to have remembered.

I wait while they process this.

"TOA" shouts one, "we measured the horizontal distance, the adjacent side, and the angle to the top of the tree, to calculate the opposite side, which is the tree height" he finishes exultantly with appreciative nods from some of the others.

"Excellent" I smile, and attack the whiteboard.

‘ ~~SOHCAH~~ TOA ‘



Then I hand out the clinometers, one between two. The Suunto ones we use (**https://www.suunto.com/en-gb/Products/Compasses/Suunto-PM-5/Suunto-PM-5360-PC/**) are not cheap, but as you will read below, they are perfect for my purposes (I have no financial connection to Suunto, or any other forestry equipment supplier).

"Look through the lens and think about what you see."

Whilst they take turns to do this, I rattle on.

"Hopefully you remember how the maths had been done for you on the diameter at breast height (dbh) tape, so that while one side showed centimetres and metres, the other side looked like inches, but was actually circumference divided by π to give a measure of diameter." (Weatherall, 2019).

They nod enthusiastically. They really like it when I tell them stuff without finishing with a question.

"Now, through the lens on the clinometer, you can see what looks like another tape measure with two scales on it, not on opposite sides, but one on the right, one on the left. What are they?" I finish.

"One side is like a protractor?"

"Like a protractor?" I query, I have a 12-year-old son, I have played this 'like' game many, many times before.

"It is a protractor." This answer may well be from a student who did geography and has used a gun clinometer. I consider, but reject, telling them that forestry is applied geography. I usually save that for another lecture.

"Yes." I smile again. "A protractor that swings with gravity, so that when you look at different angles it automatically tells you the angle you are looking at." This is good, they begin to relax. "Which side of the tape does that?"

Silence.

"If one side is a protractor," I ask, "what will you read when you hold it dead level?'

"Zero" answer two or three immediately.

But then one, who has tried it before answering, says plaintively:

"But both sides show zero when you hold it level."

There is a flurry of activity while the rest check and confirm this (Fig. 1).

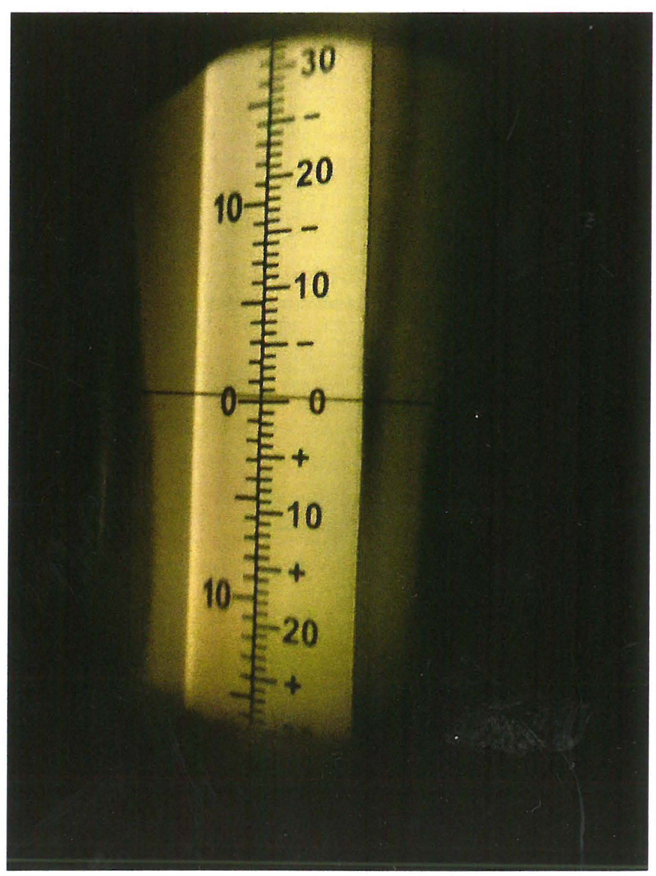


Fig. 1 View of scales inside a clinometer

"Okay," I say, "what will you read on the protractor scale side when you look vertically upwards?"

Another flurry of activity.

"Or vertically downwards" I continue which keeps them busy.

"The left-hand side is the protractor, because it shows ninety degrees when you look straight up and straight down."

"Very good" I smile again, and they look most pleased with themselves.

"So, what is the right-hand scale?"

Another flurry of activity while they look through the lens of the clinometers again. No immediate answers though.

"If you look at each end of the scale you can see a percentage sign '%'. What is it a percentage of?"

I wait as long as I think I can hold them, grappling with this, and eventually a student, usually one of those who attended a Visit Day, albeit six months to a year previously, pieces together this lecture with what they actually did.

"It is a percent of the horizontal distance that we measured out from the tree."

"Correct." Palpable sighs of relief. They have survived the interrogation, or so they think.

"How do you calculate that then?"

And this is where a student who is good at maths takes a deep breath and gets their chance to shine:

"The angles measured on the left hand-side scale in the clinometer have been converted with tangent to percentages of the horizontal distance".

"Absolutely correct."

I can see that some of them still look confused. I prove it to them by showing on the whiteboard and with the calculators on their mobile phones that in a 1 by 1 by square root 2 right-angled triangle, the tangent must be 45 degrees. Then I get them to look at 45 degrees on the left-hand scale in the clinometer and, as if by magic, they see that the right-hand scale shows 100% (Fig. 2). The opposite (or tree stem) is 100% of the adjacent (horizontal distance from the tree).

Once they all look reasonably confident that they understand, I explain that because our eyes are not at ground level, we also need to add our individual height (up to our eyes) to our answer. However, I continue quickly, to explain that as we are not always stood on level ground, it is actually better to make two right-angled triangles with the tree, one measuring up the stem and one measuring down to ground level, then sum the percentages measured on the right-hand scale in the

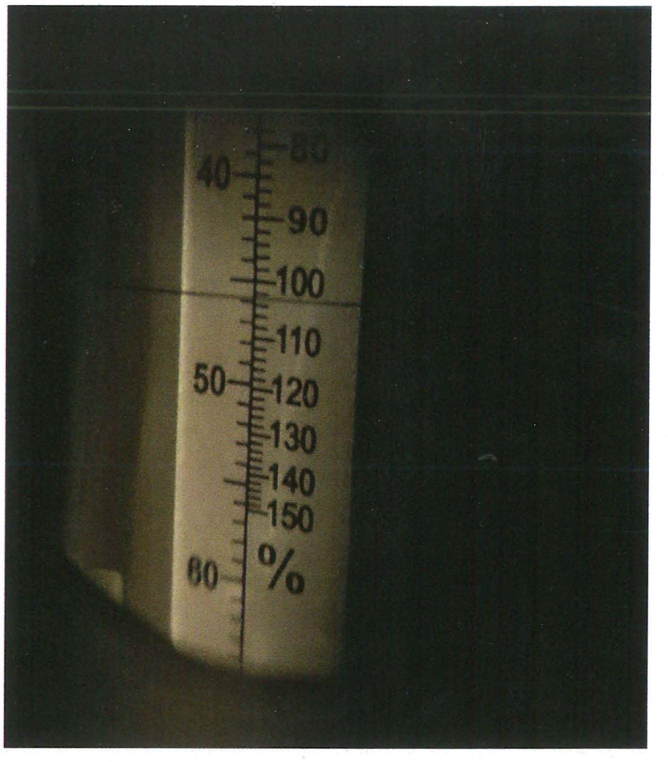


Fig. 2 100% of horizontal distance = 45 degrees

clinometer before using with the horizontal distance. I leave the challenges of what happens when the tree is growing at an angle, when we are downslope from it, or when we cannot see the true top of it, for another day. There is only one more thing to learn today:

"Shall we go outside and try it."

I may have mentioned in a previous article that I did not become a forestry lecturer to spend time indoors and our students did not choose our courses just to study inside.

Fig. 3 Using a clinometer

We are fortunate to be based at our Ambleside Campus, in the heart of the Lake District National Park. It is our best lecture room and we have plenty of trees to practise on (Fig. 3).

I think this is a perfect application of Pythagoras' theorem and SOHCAHTOA, but I am a forestry lecturer, you would expect me to think that. The question is, would it help maths teachers to use clinometers to measure tree heights to help secondary school pupils realize that maths is not just classroom theories, but underpins real world working practices?

References

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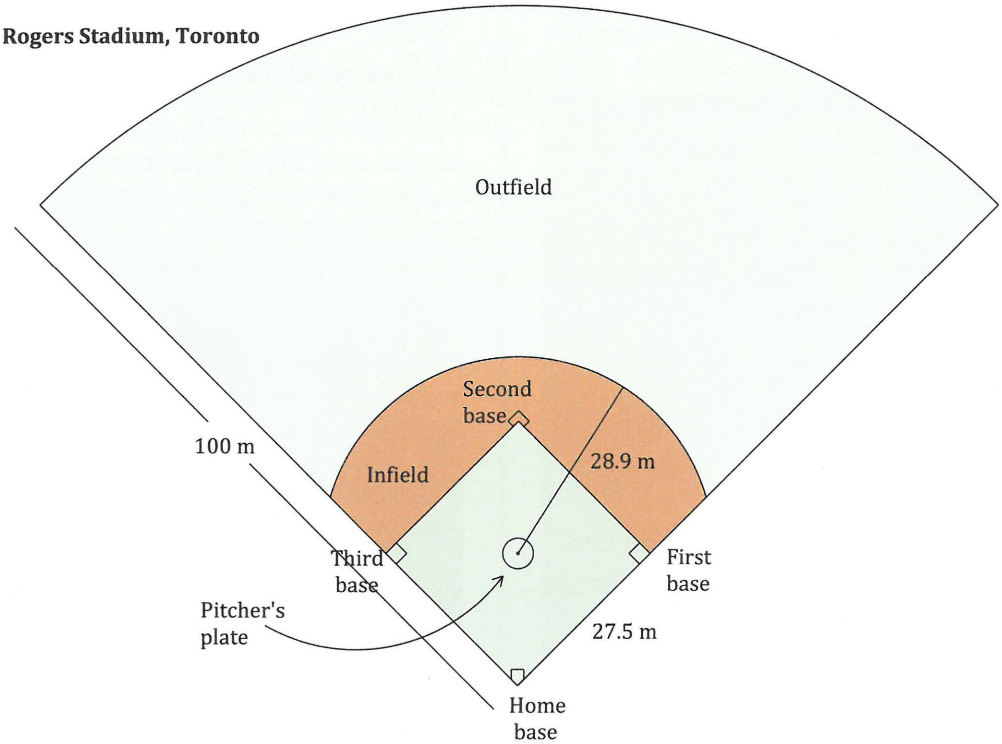
The Baseball Field

by **Chris Pritchard**

Sports pitches, courts, rings, tracks and fields provide a wealth of stimulating, real-world contexts in which to set mathematics questions. One of the lesser-known surfaces, at least in Britain, is the baseball field, which often takes the shape of a quadrant. The 'infield' is the term used for either the grassed square or the square plus the partially-surrounding dirt area. Here we will use the latter version. Whilst the dimensions of the infield are standardized, the size of the grassed outfield varies from stadium to stadium. The field at the Rogers Stadium in Toronto consists of a quadrant of radius 100 m. The arc marking the division between the infield and outfield (the 'grass line') is centred on the pitcher's plate and has radius 28.9 m.

Rogers Stadium, Toronto



Questions might include:

1. What is the perimeter of the field?
2. What is the length of the grass line?
3. What is the distance between the home base and the second base?
4. What is the shortest distance between the grass line and the outer quadrant arc?
5. What is the area of the field?
6. What are the areas of the infield and outfield?
7. What is the area not grassed?

Answers will appear in the next issue.

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